

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON SATURDAY 24th JANUARY 2026)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f(x) = \int \frac{7x^{10} + 9x^8}{(1+x^2+2x^9)^2} dx$, $x > 0$,

$\lim_{x \rightarrow 0} f(x) = 0$ and $f(1) = \frac{1}{4}$. If

$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & f'(1) & 1 \\ \alpha^2 & 4 & 1 \end{bmatrix}$ and $B = \text{adj}(\text{adj } A)$ be such

that $|B| = 81$, then α^2 is equal to

- (1) 2 (2) 3
(3) 1 (4) 4

Ans. (4)

Sol. $f(x) = \int \left(\frac{7}{x^8} + \frac{9}{x^{10}} \right) dx$
 $\left(\frac{1}{x^9} + \frac{1}{x^7} + 2 \right)^2$

Put $t = \frac{1}{x^9} + \frac{1}{x^7} + 2 \Rightarrow \frac{dt}{dx} = \frac{-9}{x^{10}} - \frac{7}{x^8}$

$f(x) = \int \frac{-dt}{t^2} = \frac{1}{t} + C$

$f(x) = \frac{1}{\frac{1}{x^9} + \frac{1}{x^7} + 2} + C$

$= \frac{x^9}{1+x^2+2x^9} + C$

Given $f(1) = \frac{1}{4} = \frac{1}{4} + C \Rightarrow C = 0$

$f(x) = \frac{x^9}{1+x^2+2x^9}$

$f'(x) = \frac{(1+x^2+2x^9) - 9x^8 - x^9(2x+18x^8)}{(1+x^2+2x^9)^2}$

$f'(x) = \frac{36-20}{16} = 1$

$A = \begin{bmatrix} 0 & 0 & 1 \\ 4 & 1 & 1 \\ \alpha^2 & \frac{1}{4} & 1 \end{bmatrix}$

$|A| = |1 - \alpha^2| = 3$

$1 - \alpha^2 = 3, -3 \Rightarrow \alpha^2 = -2, 4$

Value of $\alpha^2 = 4$

$B = \text{adj}(\text{adj } A)$

$|B| = 81 = |A|^4 \Rightarrow |A| = 3$

2. Let the length of the latus rectum of an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ($a > b$), be 30. If its eccentricity is the

maximum value of the function $f(t) = -\frac{3}{4} + 2t - t^2$,

then $(a^2 + b^2)$ is equal to -

- (1) 516 (2) 256
(3) 496 (4) 276

Ans. (3)

Sol. $f(t) = -\frac{3}{4} + 2t - t^2$

$f(t) \Big|_{\text{maximum}} = \frac{1}{4} = e \Rightarrow e^2 = \frac{1}{16} \Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{16} \dots (1)$

$\therefore \frac{2b^2}{a} = 30 \Rightarrow b^2 = 15a \dots (2)$

By (1) & (2)

$16(a^2 - 15a) = a^2 \Rightarrow 15a^2 - 16 \times 15a = 0$

$a = 16$

$b^2 = 240$

$a^2 + b^2 = 256 + 240$

$= 496$



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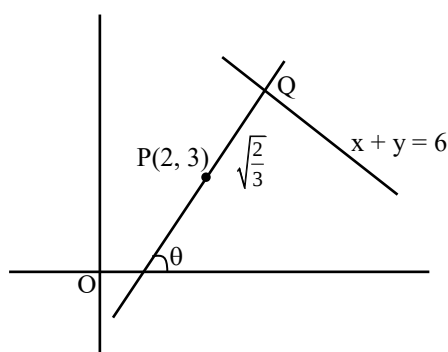
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3. Let the angles made with the positive x-axis by two straight lines drawn from the point $P(2, 3)$ and meeting the line $x + y = 6$ at a distance $\sqrt{\frac{2}{3}}$ from the point P be θ_1 and θ_2 . Then the value of $(\theta_1 + \theta_2)$ is :

- (1) $\frac{\pi}{12}$ (2) $\frac{\pi}{6}$
(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. (3)

Sol.



Let Q is $\left(\sqrt{\frac{2}{3}} \cos \theta + 2, \sqrt{\frac{2}{3}} \sin \theta + 3\right)$

so, $x + y = 6$

$$\sqrt{\frac{2}{3}} (\cos \theta + \sin \theta) + 5 = 6$$

$$\sin \theta + \cos \theta = \sqrt{\frac{3}{2}}$$

$$1 + \sin 2\theta = \frac{3}{2}$$

$$\sin 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12} \text{ \& \; } \frac{5\pi}{6}$$

$$\text{So } \theta_1 + \theta_2 = \frac{\pi}{2}$$

4. Let $\vec{a} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$. Let $\vec{v}$ be the vector in the plane of the vectors $\vec{a}$ and $\vec{b}$, such that the length of its projection on the vector $\vec{c}$ is $\frac{1}{\sqrt{14}}$. Then $|\vec{v}|$ is equal to

- (1) $\frac{\sqrt{21}}{2}$ (2) 13
(3) $\frac{\sqrt{35}}{2}$ (4) 7

Ans. (Bonus)

NTA (3)

Sol. $\vec{v} = x\vec{a} + y\vec{b} = x(2\hat{i} - \hat{j} - \hat{k}) + y(\hat{i} + 3\hat{j} - \hat{k})$

$$\vec{v} = (2x + y)\hat{i} + (3y - x)\hat{j} + (-x - y)\hat{k}$$

$$\frac{|\vec{v} \cdot \vec{c}|}{|\vec{c}|} = \frac{1}{\sqrt{14}}$$

$$\vec{v} \cdot \vec{c} = 2(2x + y) + 3y - x - 3x - 3y = 2y$$

$$\left| \frac{2y}{\sqrt{14}} \right| = \frac{1}{\sqrt{14}} \Rightarrow |2y| = 1$$

$$|\vec{v}| = \sqrt{(2x + y)^2 + (3y - x)^2 + (x + y)^2}$$

$$= \sqrt{6x^2 + 11y^2 + 4xy - 6xy + 2xy}$$

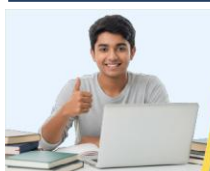
$$= \sqrt{6x^2 + \frac{11}{4}} = \frac{\sqrt{24x^2 + 11}}{2}$$

Now if we take $x^2 = 1$ then option $\frac{\sqrt{35}}{2}$ matches most probably NTA thought could be this.

5. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be an A.P. of four terms such that each term of the A. P. and its common difference l are integers. If $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 48$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4 + l^4 = 361$ then the largest term of the A.P. is equal to

- (1) 27 (2) 24 (3) 21 (4) 23

Ans. (1)



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Sol. a_1, a_2, a_3, a_4 as $a-3d, a-d, a+d, a+3d$

where $d = \frac{\ell}{2}$

$$\therefore a_1 + a_2 + a_3 + a_4 = 48 \Rightarrow 4a = 48 \Rightarrow a = 12$$

$$\& a_1 a_2 a_3 a_4 + \ell^4 = 361 \Rightarrow (a^2 - 9d^2)(a^2 - d^2) + 16d^4 = 361$$

$$\Rightarrow (144 - 9d^2)(144 - d^2) + 16d^4 = 361$$

$$\Rightarrow 25d^4 - 1440d^2 + (144)^2 = 361$$

$$(5d^2 - 144)^2 = 19^2$$

$$\therefore 5d^2 - 144 = 19 \text{ or } -19$$

$$d^2 = \frac{163}{5} \text{ or } d^2 = \frac{125}{5} = 25$$

$$d = \sqrt{\frac{163}{5}} \text{ or } d = 5$$

$$\therefore \ell = 2\sqrt{\frac{163}{5}} \text{ or } \ell = 10$$

(rejected)

$\therefore$ common difference is an integer

$$\therefore \text{largest term} = 12 + 15 = 27$$

6. Let the image of parabola $x^2 = 4y$, in the line $x - y = 1$ be $(y + \alpha)^2 = b(x - c)$, $a, b, c \in \mathbb{N}$. Then $a + b + c$ is equal to

(1) 12 (2) 4

(3) 6 (4) 8

Ans. (3)

Sol. Parametric point P on $x^2 = 4y$ is $P(2t, t^2)$

$\therefore$ mirror image of P in $x - y = 1$ is

$$Q \equiv \left(2t - \frac{2 \cdot 1 \cdot (2t - t^2 - 1)}{2}, t^2 + \frac{2 \cdot 2(1) \cdot (2t - t^2 - 1)}{2} \right)$$

$$Q \equiv (t^2 + 1, 2t - 1) \equiv (h, k)$$

$\therefore$ locus of Q is $x = \frac{(y+1)^2}{4} + 1$ which is the required parabola.

$$\therefore (y+1)^2 = 4(x-1)$$

$$\therefore a = 1, b = 4, c = 1$$

$$\therefore a + b + c = 6$$

7. $\left(\frac{1}{3} + \frac{4}{7} \right) + \left(\frac{1}{3^2} + \frac{1}{3} \times \frac{4}{7} + \frac{4^2}{7^2} \right) +$
 $\left(\frac{1}{3^3} + \frac{1}{3^2} \times \frac{4}{7} + \frac{1}{3} \times \frac{4^2}{7^2} + \frac{4^3}{7^3} \right) + \dots$ upto infinite terms is equal to -

(1) $\frac{5}{2}$ (2) $\frac{7}{4}$ (3) $\frac{4}{3}$ (4) $\frac{6}{5}$

Ans. (1)

Sol. Let $a = \frac{4}{7}$, $b = \frac{1}{3}$

$$\text{Multiply } N^r \text{ and } D^r \text{ by } (a - b) = \frac{4}{7} - \frac{1}{3} = \frac{5}{21}$$

$$\frac{1}{a-b} [(a^2 - b^2) + (a^3 - b^3) + (a^4 - b^4) + \dots \infty]$$

$$\frac{1}{a-b} \left[\frac{a^2}{1-a} - \frac{b^2}{1-b} \right] = \frac{21}{5} \left[\frac{\frac{16}{49}}{1-\frac{4}{7}} - \frac{\frac{1}{9}}{1-\frac{1}{3}} \right]$$

$$= \frac{21}{5} \left[\frac{16}{21} - \frac{1}{6} \right] = \frac{21}{5} \left[\frac{96-21}{21 \cdot 6} \right]$$

$$= \frac{75}{5 \cdot 6} = \frac{15}{6} = \frac{5}{2}$$

8. Let $P = [p_{ij}]$ and $Q = [q_{ij}]$ be two square matrices of order 3 such that $q_{ij} = 2^{(i+j-1)} p_{ij}$ and $\det(Q) = 2^{10}$. Then the value of $\det(\text{adj}(\text{adj } P))$ is :

(1) 32

(2) 16

(3) 81

(4) 124

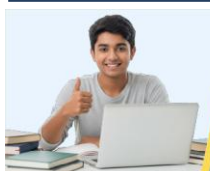
Ans. (2)

Sol. $\begin{vmatrix} 2p_{11} & 2^2 p_{12} & 2^3 p_{13} \\ 2^2 p_{21} & 2^3 p_{22} & 2^4 p_{23} \\ 2^3 p_{31} & 2^4 p_{32} & 2^5 p_{33} \end{vmatrix} = 2^{10}$

$$2^2 \cdot 2 \cdot 2^3 \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{21} & 2p_{22} & 2p_{23} \\ 2^2 p_{31} & 2^2 p_{32} & 2^2 p_{33} \end{vmatrix} = 2^{10}$$

$$2^9 \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix} = 2^{10} \Rightarrow |P| = 2$$

$$|\text{adj}(\text{adj}(P))| = |P|^{(n-1)^2} = |P|^4 = 2^4 = 16$$



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9. The letters of the word "UDAYPUR" are written in all possible ways with or without meaning and these words are arranged as in a dictionary. The rank of the word "UDAYPUR" is :

- (1) 1580 (2) 1578
(3) 1579 (4) 1581

Ans. (1)

Sol. ADIPRUU

$$A \rightarrow \frac{6!}{2!} = 360$$

$$D \rightarrow \frac{6!}{2!} = 360$$

$$P \rightarrow \frac{6!}{2!} = 360$$

$$R \rightarrow \frac{6!}{2!} = 360$$

$$UA \rightarrow 5! = 120$$

$$UDAP \rightarrow 3! = 6$$

$$UDAR \rightarrow 3! = 6$$

$$UDAU \rightarrow 3! = 6$$

$$UDAYPRU \rightarrow 1$$

$$UDAYPUR \rightarrow 1$$

$$\text{Total} = 1580$$

10. The largest value of n, for which 40^n divides $60!$, is

- (1) 13 (2) 11
(3) 12 (4) 14

Ans. (4)

Sol. $40^n = 2^{3n} \times 5^n$

$$E_2(60!) = \left[\frac{60}{2} \right] + \left[\frac{60}{2^2} \right] + \left[\frac{60}{2^3} \right] + \left[\frac{60}{2^4} \right] + \left[\frac{60}{2^5} \right]$$

$$= 30 + 15 + 7 + 3 + 1 = 56$$

$$E_5(60!) = \left[\frac{60}{5} \right] + \left[\frac{60}{5^2} \right]$$

$$= 12 + 2 = 14$$

$$40^n = (2^3)^n \times 5^n = (2^3 \times 5)^n$$

$$60! = 2^{56} \times 5^{14} \dots = 2^{14} \cdot (2^3 \cdot 5)^{14}$$

$\therefore$ Maximum value of n is 14.

11. The sum of all values of α , for which the shortest distance between the lines

$$\frac{x+1}{\alpha} = \frac{y-2}{-1} = \frac{z-4}{-\alpha} \text{ and } \frac{x}{\alpha} = \frac{y-1}{2} = \frac{z-1}{2\alpha} \text{ is } \sqrt{2}, \text{ is}$$

- (1) 8 (2) -6
(3) 6 (4) -8

Ans. (2)

$$\text{Sol. } \sqrt{2} = \frac{\begin{vmatrix} -1 & 1 & 3 \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}$$

$$\sqrt{2} = \frac{-1(-2\alpha + 2\alpha) - 1(2\alpha^2 + \alpha^2) + 3(2\alpha + \alpha)}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & -1 & -\alpha \\ \alpha & 2 & 2\alpha \end{vmatrix}}$$

$$\sqrt{2} = \frac{-3\alpha^2 + 9\alpha}{\sqrt{9\alpha^4 + 9\alpha^2}}$$

$$\sqrt{2} = \frac{-\alpha + 3}{\sqrt{\alpha^2 + 1}}$$

$$\Rightarrow 2\alpha^2 + 2 = \alpha^2 + 9 - 6\alpha$$

$$\alpha^2 + 6\alpha - 7 = 0$$

$$(\alpha + 7)(\alpha - 1) = 0$$

$$\alpha = -7, 1$$

$$\text{sum} = -7 + 1 = -6$$

option (2)

12. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by $x = 0$, $x = 1$, $y^2 = x$ and $y = |\alpha x - 5| - |1 - \alpha x| + \alpha x^2$. Then $(f(0) + f(1))$ is equal to

- (1) 9 (2) 14
(3) 7 (4) 12

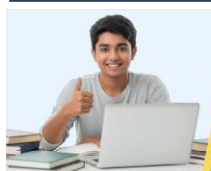
Ans. (3)

Sol. at $\alpha = 0 \Rightarrow f(0)$

$$x = 0, x = 1, y^2 = x$$

$$y = |0 \cdot x - 5| - |1 - 0 \cdot x| + 0 \cdot x^2$$

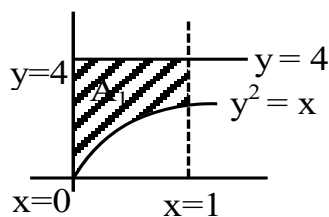
$$y = 4$$



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$$A_1 = \int_0^1 (4 - \sqrt{x}) dx$$

$$= 4x - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \bigg|_0^1$$

$$= 4 - \frac{2}{3}(1) = \frac{10}{3}$$

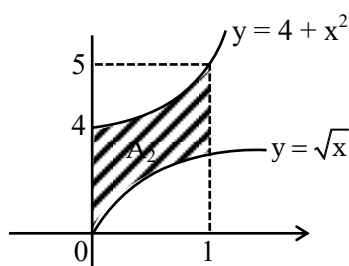
at $\alpha = 1 \Rightarrow f(1)$

$x = 0, x = 1, y^2 = x,$

$y = |x - 5| - |1 - x| + x^2$ in $x \in (0, 1)$

$y = 5 - x - (1 - x) + x^2$

$y = 4 + x^2$



$$A_2 = \int_0^1 ((4 + x^2) - (\sqrt{x})) dx$$

$$= 4x + \frac{x^3}{3} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \bigg|_0^1$$

$$= 4 + \frac{1}{3} - \frac{2}{3} = \frac{11}{3}$$

$$|f(0) + f(1)| = |A_1 + A_2| = \left| \frac{10}{3} + \frac{11}{3} \right| = \left| \frac{21}{3} \right| = 7$$

option (3)

13. If the domain of the function $f(x) = \sin^{-1} \left(\frac{1}{x^2 - 2x - 2} \right)$, is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then

$\alpha + \beta + \gamma + \delta$ is equal to

- (1) 2 (2) 4
(3) 3 (4) 5

Ans. (2)

Sol. $-1 \leq \frac{2}{x^2 - 2x - 2} \leq 1$

$$\frac{1 + x^2 - 2x - 2}{x^2 - 2x - 2} \geq 0 \Rightarrow \frac{(x-1)^2 - 2}{(x-1)^2 - 3} \geq 0$$

$$\Rightarrow \frac{(x-1-\sqrt{2})(x-1+\sqrt{2})}{(x-1-\sqrt{3})(x-1+\sqrt{3})} \geq 0$$

$$x \in (-\infty, 1-\sqrt{3}) \cup [1-\sqrt{2}, 1+\sqrt{2}] \cup (1+\sqrt{3}, \infty) \dots (1)$$

$$1 - \frac{1}{x^2 - 2x - 2} \geq 0 \Rightarrow \frac{x^2 - 2x - 3}{x^2 - 2x - 2} \geq 0$$

$$\Rightarrow \frac{(x+1)(x-3)}{(x-1+\sqrt{3})(x-1-\sqrt{3})} \geq 0$$

$$x \in (-\infty, -1] \cup (1-\sqrt{3}, \sqrt{3}+1) \cup [3, \infty) \dots (2)$$

$$(1) \cap (2)$$

$$\Rightarrow x \in (-\infty, -1] \cup [1-\sqrt{2}, 1+\sqrt{2}] \cup [3, \infty)$$

$$\therefore \alpha + \beta + \gamma + \delta = 4$$

14. Let $X = \{x \in \mathbb{N} : 1 \leq x \leq 19\}$ and for some $a, b \in \mathbb{R}$,

$Y = \{ax + b : x \in X\}$. If the mean and variance of the elements of Y are 30 and 750, respectively, then the sum of all possible values of b is

- (1) 20 (2) 80
(3) 100 (4) 60

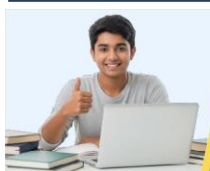
Ans. (4)

Sol. $\Sigma y_i = a \Sigma x_i + \Sigma b$

$$= a \times (1 + 2 + \dots + 19) + 19b$$

$$\frac{\Sigma y_i}{19} = \frac{a \times 19 \times 20}{2 \times 19} + b$$

$$30 = 10a + b \dots (1)$$



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$$\text{Variance of } X = \frac{\sum x_i^2}{19} - \left(\frac{\sum x_i}{19} \right)^2$$

$$= \frac{19 \times 20 \times 39}{19 \times 6} - (10)^2 = 30$$

Variance of $Y = a^2$ (variance of X)

$$750 = a^2 \times 30$$

$$a^2 = 25 \Rightarrow a = \pm 5$$

$$\text{if } a = +5 \Rightarrow b = 30 - 50 = -20 \quad \dots \text{from (i)}$$

$$\text{if } a = -5 \Rightarrow b = 30 + 50 = 80 \quad \dots \text{from (i)}$$

$$\text{sum of values of } b = 80 - 20 = 60$$

options (4)

15. Consider the following three statements for the function $f: (0, \infty) \rightarrow \mathbb{R}$ defined by

$$f(x) = |\log_e x| - |x - 1|:$$

(I) f is differentiable at all $x > 0$.

(II) f is increasing in $(0, 1)$.

(III) f is decreasing in $(1, \infty)$.

Then.

(1) All (I), (II) and (III) are TRUE.

(2) Only (I) is TRUE.

(3) Only (II) and (III) are TRUE.

(4) Only (I) and (III) are TRUE.

Ans. (4)

Sol. $f(x) = |\ln x| - |x - 1|$

$$= \begin{cases} \ln x - (x - 1) & x \geq 1 \\ -\ln x + (x - 1) & 0 < x < 1 \end{cases}$$

$$= \begin{cases} \ln x - x + 1 & x \geq 1 \\ -\ln x + x - 1 & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x} - 1 & x \geq 1 \\ -\frac{1}{x} + 1 & 0 < x < 1 \end{cases}$$

$$f'(1^+) = f'(1^-) = 0 \Rightarrow f(x) \text{ is differentiable } \forall x > 0$$

$$f'(x) < 0 \quad \forall x > 1$$

$$f'(x) < 0 \quad \forall 0 < x < 1$$

$$\Rightarrow f(x) \text{ is decreasing } \forall x \in (0, \infty)$$

Option (4)

16. Let $y = y(x)$ be a differentiable function in the interval $(0, \infty)$ such that $y(1) = 2$.

$$\text{and } \lim_{t \rightarrow x} \left(\frac{t^2 y(x) - x^2 y(t)}{x - t} \right) = 3 \quad \text{for each } x > 0.$$

Then $2y(2)$ is equal to

(1) 18

(2) 23

(3) 27

(4) 12

Ans. (2)

Sol. $\lim_{t \rightarrow x} \frac{2t f(x) - x^2 f'(t)}{-1} = 3$

$$x^2 f'(x) - 2x f(x) = 3$$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{3}{x^2}$$

$$\text{I.F.} = e^{-\int \frac{2}{x} dx} = e^{-2 \log_e x} = 1/x^2$$

$$y \cdot \frac{1}{x^2} = \int \frac{3}{x^4} dx$$

$$\frac{y}{x^2} = -\frac{1}{x^3} + c \Rightarrow y = cx^2 - \frac{1}{x} = f(x)$$

$$f(1) = 2 = c - 1 \Rightarrow c = 3$$

$$f(x) = 3x^2 - \frac{1}{x}$$

$$f(2) = 12 - \frac{1}{2} \Rightarrow 2f(2) = 23$$

17. Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x$,

$$x \neq 0, \text{ where } m = \sum_{i=1}^9 (i)^2. \text{ Then } f(5) - f(2) \text{ is equal}$$

to

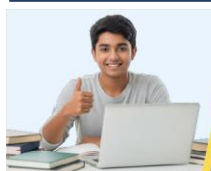
(1) -9

(2) 36

(3) 18

(4) 9

Ans. (3)



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Sol. $m = \frac{9 \times 10 \times 19}{6} = 15 \times 19$

$$3f(x) + 2f\left(\frac{15}{x}\right) = 5x$$

Replace x by $\frac{15}{x}$

$$3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x}$$

$$9f(x) - 4f(x) = 15x - \frac{150}{x}$$

$$5f(x) = 15x - \frac{150}{x}$$

$$f(x) = 3x - \frac{30}{x}$$

$$f(5) = 15 - \frac{30}{5} = 9$$

$$f(2) = 6 - 15 = -9$$

$$f(5) - f(2) = 18$$

- 18.** The smallest positive integral value of a , for which all the roots of $x^4 - ax^2 + 9 = 0$ are real and distinct, is equal to

- (1) 9 (2) 3
(3) 4 (4) 7

Ans. (4)

Sol. $x^4 - ax^2 + 9 = 0 \quad \dots(1)$

let $x^2 = t$

$$t^2 - at + 9 = 0 \quad \dots(2)$$

for roots of equation (1) to be real & distinct roots of equation (2) must be positive & distinct.

(i) $D > 0 \Rightarrow a^2 - 36 > 0 \Rightarrow a \in (-\infty, -6) \cup (6, \infty)$

(ii) $\frac{-b}{2a} > 0 \Rightarrow \frac{a}{2} > 0 \Rightarrow a > 0$

(iii) $f(0) > 0 \Rightarrow 9 > 0 \Rightarrow a \in \mathbb{R}$

By (i) $\cap$ (ii) $\cap$ (iii)

$\therefore a \in (6, \infty)$

$\therefore$ least integral value of a is 7

- 19.** Let $\vec{a} = 2\hat{i} - 5\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$. If $\vec{c}$ is a vector such that

$$2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0} \quad \text{and} \quad (\vec{a} - \vec{b}) \cdot \vec{c} = -97, \text{ then}$$

$|\vec{c} \times \vec{k}|^2$ is equal to

- (1) 193 (2) 233
(3) 218 (4) 205

Ans. (3)

Sol. $2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = \vec{0}$

$$\Rightarrow (2\vec{a} + 3\vec{b}) \times \vec{c} = \vec{0} \Rightarrow \vec{c} = \lambda(2\vec{a} + 3\vec{b})$$

$$\Rightarrow \vec{c} = \lambda(7\hat{i} - 13\hat{j} + 19\hat{k})$$

$$\text{Now } (\vec{a} - \vec{b}) \cdot \vec{c} = \lambda(7 + 52 + 38) + 97\lambda = -97$$

$$\Rightarrow \lambda = -1$$

$$\text{Now } \vec{c} = -7\hat{i} + 13\hat{j} - 19\hat{k}$$

$$\Rightarrow \vec{c} \times \vec{k} = 7\hat{j} + 13\hat{i} \Rightarrow |\vec{c} \times \vec{k}|^2 = 7^2 + 13^2 = 218$$

- 20.** Let $[t]$ denote the greatest integer less than or equal to t . If the function

$$f(x) = \begin{cases} b^2 \sin\left(\frac{\pi}{2} \left[\frac{\pi}{2} (\cos x + \sin x) \cos x \right] \right) & , x < 0 \\ \frac{\sin x - \frac{1}{2} \sin 2x}{x^3} & , x > 0 \\ a & , x = 0 \end{cases}$$

is continuous at $x = 0$, then $a^2 + b^2$ is equal to

- (1) $\frac{5}{8}$ (2) $\frac{9}{16}$
(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. (3)

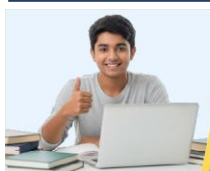
Sol. $f(0) = a$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{\sin x(1 - \cos x)}{x^3} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} \left[b^2 \sin \frac{\pi}{2} \left[\frac{\pi}{2} (\sin x + \cos x) \cos x \right] \right] = b^2$$

$$\therefore a = \frac{1}{2} \text{ \& } b^2 = \frac{1}{2}$$

$$\text{so } (a^2 + b^2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$



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SECTION-B

21. If $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 (y + xe^x) f(y) dy$, then $e + f(0)$ is equal to _____.

Ans. (2)

Sol. $f(x) = e^x + \int_0^1 yf(y)dy + xe^x \int_0^1 f(y)dy$
 $f(x) = e^x + A + Bxe^x$
 $A = \int_0^1 yf(y) dy = \int_0^1 y(A + e^y + By e^y) dy$
 $A = \frac{A}{2} + (0 - (-1)) + B(e - 1)$
 $\frac{A}{2} + B(1 - e) = 1$
 $B = \int_0^1 f(y) dy$
 $B = \int_0^1 (e^y + A + By e^y) dy$
 $B = (e - 1) + A + B(0 - (-1))$
 $B = e - 1 + A + B \Rightarrow A = 1 - e$
 $f(x) = e^x + A + Bxe^x$
 $f(0) = 1 + A = 1 - e + 1 = 2 - e$
 $e + f(0) = 2$

22. Let (h, k) lie on the circle $C : x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e . Then the value of $\frac{5}{e^2}$ is equal to _____.

Ans. (9)

Sol. Let $P \equiv (2\cos \theta, 2\sin \theta)$
 $\therefore$ coordinates of $Q = (4\cos\theta + 1, 6\sin\theta + 3)$
 $\therefore$ locus of Q is $\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-3}{6}\right)^2 = 1$
 $\therefore e^2 = 1 - \frac{16}{36} = \frac{5}{9}$
 $\therefore \frac{5}{e^2} = 9$

23. Let $z = (1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni)$, where $i = \sqrt{-1}$. If $|z|^2 = 44200$, then n is equal to _____.

Ans. (5)

Sol. $|z|^2 = 2^3 \cdot 5^2 \cdot 13 \cdot 17$
 $\prod_{r=1}^n (1+r^2) = 2^3 \cdot 5^2 \cdot 13 \cdot 17 = (2) \cdot (5) \cdot (2.5) \cdot (17) \cdot (2.13) = 2.5 \cdot 10 \cdot 17 \cdot 26$
 so $n = 5$

24. Let S be a set of 5 elements and $P(S)$ denote the power set of S . Let E be an event of choosing an ordered pair (A, B) from the set $P(S) \times P(S)$ such that $A \cap B = \emptyset$. If the probability of the event E is $\frac{3^p}{2^q}$, where $p, q \in \mathbb{N}$, then $p + q$ is equal to _____.

Ans. (15)

Sol. $S = \{a, b, c, d, e\}$
 $P(S)$ contains 32 elements
 both set A and set B are subsets of $P(S)$
 Every element has 4 choices

A	B
✓	✓
✓	x
x	✓
x	x

Favourable cases = 3^5

Total cases = 4^5

$$P = \frac{3^5}{4^5} = \frac{3^5}{2^{10}}$$

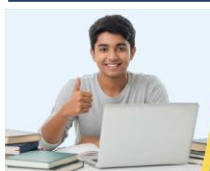
$$m = 5, n = 10$$

$$m + n = 15$$

25. The number of elements in the set $\left\{ x \in [0, 180^\circ] : \tan(x + 100^\circ) = \frac{\tan(x + 50^\circ) \tan x \tan(x - 50^\circ)}{\tan x} \right\}$ is _____.

Ans. (4)

Sol. $\frac{\tan(x + 100^\circ)}{\tan x} = \tan(x + 50^\circ) \tan(x - 50^\circ)$
 $\frac{\sin(x + 100^\circ) \cos x}{\cos(x + 100^\circ) \sin x} = \frac{\sin(x + 50^\circ) \sin(x - 50^\circ)}{\cos(x + 50^\circ) \cos(x - 50^\circ)}$
 Apply C & D
 $\frac{\sin(2x + 100^\circ)}{\sin 100^\circ} = \frac{\cos 100^\circ}{-\cos 2x}$
 $2\sin(2x + 100^\circ) \cos 2x + \sin 200^\circ = 0$
 $\sin(4x + 100^\circ) + \sin 100^\circ + \sin 200^\circ = 0$
 $\sin(4x + 100^\circ) = -2\sin 150^\circ \cos 50^\circ$
 $\sin(4x + 100^\circ) = -\cos 50^\circ = \sin(-40^\circ)$
 $\therefore 4x + 100^\circ = n\pi + (-1)^n \cdot (-40^\circ)$
 $x = \frac{n\pi + (-1)^{n+1}(40^\circ) - 100^\circ}{4}$
 $\therefore x = 30^\circ, 55^\circ, 120^\circ, 145^\circ$ in $(0, \pi)$
 $\therefore$ no. of solutions = 4



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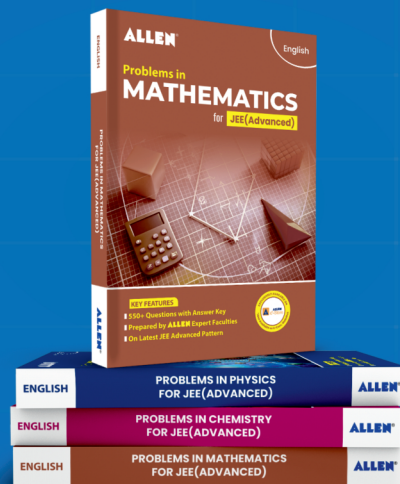
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